LexiFi pricing models and methods

LexiFi

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1 Equities

All equity models support discrete and continuous dividends, with a term structure in the latter case, and quanto adjustments.
Note that for the sake of simplicity, all model dynamic will be written assuming no dividends.

1.1 Static replication

The replication price is:

$$Price(t,T,S) = \gamma ZC(t,T) + \sum_{i=1}^{n} \alpha_i Call(t,T,S,F(t,T)+i\delta K) + \sum_{j=1}^{m} \beta_j Put(t,T,S,F(t,T)-j\delta K)$$

- Available for sums of European payoff-like single-underlying products.
- Automatic decomposition.
- Quasi-Closed form solution.

1.2 Black Scholes

The dynamic of the underlying is:

$$\frac{dS(t)}{S(t)} = r(t)dt + \sigma(t)dW(t)$$

- Support of both constant and deterministic volatility term-structure.
- Monte Carlo multiple-asset or PDE single/dual-asset model implementation.
- Smart adjuster to cheaply take in account most volatility smile

1.3 Local volatility

The dynamic of the underlying is:

$$\frac{dS(t)}{S(t)} = r(t)dt + \sigma(t,S(t))dW(t)$$

- Calibration to European call and put quotes using an implied volatility surface fitting.
- Modern methods including Andreason-Huge calibration.
- Advanced time discretisation scheme: Runge Kutta, Euler or automatic switch.
- Monte Carlo multiple-asset or PDE up to three assets model implementation.

1.4 Heston (stochastic volatility)

The dynamic of the underlying is:

$$\begin{aligned}\frac{dS(t)}{S(t)} &= r(t)dt + \sqrt{V(t)}dW_S(t) \\ dV(t) &= \kappa[\theta - V(t)]dt + \eta\sqrt{V(t)}dW_V(t), V(0) = v_0 \\ dW_S(t)dW_V(t) &= \rho dt\end{aligned}$$

- Calibration to European call and put quotes using semi-closed formula and numerical integration.
- Advanced time discretization scheme (quadratic exponential).
- Monte Carlo multiple-asset or PDE single asset model implementation.

1.5 Shifted Black

The dynamic of the underlying forward is:

$$dF(t,T) = \lambda(t)[b(t)F(t,T) + (1 - b(t))F(0,t)]dW(t)$$

- Linear local volatility model.
- Calibration to European call and put quotes using semi-closed formula.
- Fit the skew of the volatility surface.
- Monte Carlo large step multiple-asset or PDE up to three assets model implementation.

1.6 Heston - Local Stochastic Volatility

The dynamic of the underlying is:

$$\begin{aligned}\frac{dS(t)}{S(t)} &= r(t)dt + \sqrt{V(t)}L(t,S(t))dW_S(t) \\ dV(t) &= \kappa[\theta - V(t)]dt + \eta\sqrt{V(t)}dW_V(t), V(0) = 1 \\ dW_S(t)dW_V(t) &= \rho dt\end{aligned}$$

- Calibration to European call and put quotes using Heston calibration, Local Volatility calibration followed by either Particle Method or Fokker-Planck equation to compute probability density.
- Possibility to give more importance to Local or Stochastic volatility part using mixing weight factor.
- Advanced time discretization scheme for the variance (quadratic exponential).
- Monte Carlo multiple-asset or PDE single asset model implementation.

1.7 Merton Jump Model

The dynamic of the underlying is:

$$\frac{dS(t)}{S(t)} = r(t)dt + \sigma(t)dW(t) + dJ(t)$$

- Jump diffusion extension of the Black-Scholes model.
- Calibration to European call and put quotes using semi-closed formula.
- Ability to fit short-term smile.
- Term structure possible for each parameter.
- Large step simulation using efficient Poisson distribution simulation.
- Monte Carlo multiple-asset or PIDE single asset model implementation.

1.8 Bates Jump Model

The dynamic of the underlying is:

$$\begin{aligned} \frac{dS(t)}{S(t)} &= r(t)dt + \sqrt{V(t)}dW_S(t) + dJ(t) \\ dV(t) &= \kappa[\theta - V(t)]dt + \eta\sqrt{V(t)}dW_V(t), V(0) = v_0 \\ dW_S(t)dW_V(t) &= \rho dt \end{aligned}$$

- Jump diffusion extension of the Heston model.
- Calibration to European call and put quotes using semi-closed formula and numerical integration.
- Ability to fit short and long-term smile.
- Advanced time discretization scheme for the variance (quadratic exponential).
- Monte Carlo multiple-asset implementation.

1.9 Rough Heston Model

The dynamic of the underlying is:

$$
\begin{aligned}
dX_i(t) &= \sqrt{V_i(t)} X_i(t) dW_i^X(t) \\
V_t &= V_0 + \frac{1}{\Gamma(\alpha)} \left(\lambda \int_0^t (t-s)^{\alpha-1} (\theta^0(s) - V_s) ds + \nu \int_0^t (u-s)^{\alpha-1} \sqrt{V_s} dW_s \right) \\
S_i(t) &= [F_i(0,t) - D_i(t)] X_i(t) + D_i(t) \\
X_i(0) &= 1 \\
dW_i^X(t) dW(t) &= \rho dt
\end{aligned}
$$

- Multi-asset model in which assets have a stochastic instantaneous variance drived by a Volterra process.
- Calibration to call and put quotes is computed using numerical integration and an approximation method to find an expression for the characteristic function.
- Ability to fit short and long-term smile.
- Advanced time discretization scheme for the variance (Hybrid Quadratic Exponential).
- Monte Carlo multiple-asset implementation (using convolution).

2 Interest rates

All interest-rate models are delivered with calibration routines for cap/floor and swaption quotes.

2.1 Static replication on Libor or CMS

The replication price is:

$$Price(t,T,S) = \gamma ZC(t,T) + \sum_{i=1}^{n} \alpha_i Call(t,T,S,F(t,T)+i\delta K) + \sum_{j=1}^{m} \beta_j Put(t,T,S,F(t,T)-j\delta K)$$

- Only available for vanilla contracts.
- Automatic decomposition.
- Convexity Adjustment.
- Shifted-SABR interpolation formula.
- Quasi-Closed form solution.

2.2 Hull-White 1 factor

The dynamic of the short rate is:

$$dr(t) = [\theta(t) - a(t)r(t)]dt + \sigma(t)dW(t)$$

- Monte Carlo large step or PDE model implementation.
- Exact large steps simulation using the forward-neutral probability.
- Support of constant, exponential and step volatility term structure.

2.3 Hull-White 2 factors (G2++)

The short rate is given by:

$$\begin{aligned} r(t) &= x(t) + y(t) + \phi(t) \\ dx(t) &= -ax(t)dt + \sigma(t)dW_x(t) \\ dy(t) &= -by(t)dt + \eta(t)dW_y(t) \\ dW_x(t)dW_y(t) &= \rho dt \end{aligned}$$

- Monte Carlo large step or PDE model implementation.
- Various calibration modes on caps and swaptions.
- Exact large steps simulation using the forward-neutral probability.
- Support of constant and step volatility term structures.

2.4 Cheyette (quasi-Gaussian model)

The short rate is given by:

$$
\begin{aligned}
r(t) &= f(0,t) + x(t) \\
dx(t) &= [y(t) - \chi(t)x(t)]dt + \sigma_r(t)dW_r(t) \\
dy(t) &= [\sigma_r(t)^2 - 2\chi(t)y(t)]dt
\end{aligned}
$$

where

$$\sigma_r(t) = \alpha(t) + \beta(t)x(t)$$

or

$$
\begin{aligned}
\sigma_r(t) &= \sqrt{z(t)}[\alpha(t) + \beta(t)x(t)] \\
dz(t) &= \theta(t)[z_0 - z(t)]dt + \eta(t)\sqrt{z(t)}dW_z(t), z(0) = z_0 = 1 \\
dW_r(t)dW_z(t) &= 0
\end{aligned}
$$

- Monte Carlo using QE scheme or PDE implementation.
- Linear local volatility and linear local volatility with CIR stochastic volatility parameterizations.
- Time-dependent parameters.
- Captures most shapes of volatility smiles.
- Optimized calibration: calibrating first a proxy swap rate market model (SMM) on implied volatility, then bootstrapping the Cheyette model parameters to fit SMM parameters.

2.5 Cheyette (stochastic local volatility model)

The short rate is given by:

$$
\begin{aligned}
r(t) &= f(0,t) + x(t) \\
dx(t) &= [y(t) - \lambda(t)x(t)]dt + \sigma_r(t)dW_r(t) \\
dy(t) &= [\sigma_r(t)^2 - 2\chi(t)y(t)]dt
\end{aligned}
$$

where $\sigma_r(t)$ can be a local volatility

$$\sigma_r(t) = \sigma_r(t, x(t), y(t))$$

or a stochastic local volatility

$$
\begin{aligned}
\sigma_r(t) &= \sqrt{z(t)}\sigma_r(t, x(t), y(t)) \\
dz(t) &= \theta(t)[z_0 - z(t)]dt + \eta(t)\sqrt{z(t)}dW_z(t) \\
dW_r(t)dW_z(t) &= 0 \\
z(0) &= z_0 = 1
\end{aligned}
$$

- Monte Carlo using QE scheme
- PDE implementation with/without y approximation (the size of the PDE is 1,2 or 3).

- Local volatility and local volatility with CIR stochastic volatility parameterizations.
- Match perfectly the volatility smile for a single swap tenor.
- Calibration with a particle method (exact calibration).

2.6 Lognormal forward-LIBOR model (LFM)

The dynamic of the forward-LIBOR are:

$$\frac{dL(t,t_{i-1},t_i)}{L(t,t_{i-1},t_i)} = \sigma_i(t)dW_i(t) \;\; under \;\; \mathbb{Q}^{t_i}$$
$$dW_i(t)dW_j(t) = \rho_{i,j}dt$$

- Monte Carlo model implementation.
- Large steps simulation (using Runge-Kutta discretization).
- Functional volatility and correlation structures.
- Dimension reduction using principal component analysis.

2.7 Lognormal forward-LIBOR + stochastic volatility model (LFM+SV)

The dynamic of the forward-LIBOR are:

$$\frac{dL(t,t_{i-1},t_i)}{L(t,t_{i-1},t_i)} = \sigma_i(t)\sqrt{V(t)}dW_i(t) \;\; under \;\; \mathbb{Q}^{t_i}$$
$$dV(t) = \theta(v_0 - V(t))dt + \eta\sqrt{V(t)}dW_V(t), V(0) = v_0 = 1$$
$$dW_i(t)dW_j(t) = \rho_{i,j}dt$$
$$dW_i(t)dW_V(t) = 0$$

- Large steps simulation (using Runge-Kutta discretization).
- Functional volatility and correlation structures.
- Calibration on Cap and Swaption skew and smile.
- Dimension reduction using principal component analysis.

2.8 Shifted-Lognormal forward-LIBOR model (SLFM)

The dynamic of the forward-LIBOR are:

$$dL(t,t_{i-1},t_i) = \sigma_i(t)[b_i(t)L(t,t_{i-1},t_i) + (1-b_i(t))L(0,t_{i-1},t_i)]dW_i(t) \;\; under \;\; \mathbb{Q}^{t_i}$$
$$dW_i(t)dW_j(t) = \rho_{i,j}dt$$

- Monte Carlo model implementation.
- Large steps simulation (using Runge-Kutta discretization).

- Functional volatility and correlation structures.
- Calibration on Cap and Swaption skew and smile.
- Dimension reduction using principal component analysis.

2.9 Shifted-Lognormal forward-LIBOR model + stochastic volatility (SLFM+SV)

The dynamic of the forward-LIBOR are:

$$
\begin{aligned}
dL(t, t_{i-1}, t_i) &= \sigma_i(t)[b_i(t)L(t, t_{i-1}, t_i) + (1 - b_i(t))L(0, t_{i-1}, t_i)]\sqrt{V(t)}dW_i(t) \;\; under \;\; \mathbb{Q}^{t_i} \\
dV(t) &= \theta(v_0 - V(t))dt + \eta\sqrt{V(t)}dW_V(t), V(0) = v_0 = 1 \\
dW_i(t)dW_j(t) &= \rho_{i,j}dt \\
dW_i(t)dW_V(t) &= 0
\end{aligned}
$$

- Large steps simulation (using Runge-Kutta discretization).
- Functional volatility and correlation structures.
- Calibration on Cap and Swaption skew and smile.
- Dimension reduction using principal component analysis.

3 Inflation

3.1 Static replication on Index and YoY

The replication price is:

$$Price(t,T,S) = \gamma ZC(t,T) + \sum_{i=1}^{n} \alpha_i Call(t,T,S,F(t,T)+i\delta K) + \sum_{j=1}^{m} \beta_j Put(t,T,S,F(t,T)-j\delta K)$$

- Only available for vanilla contracts.
- Automatic decomposition.
- Quasi-Closed form solution.

3.2 Jarrow-Yildirim

The dynamic under the nominal (cash) risk neutral probability of the inflation index is:

$$\begin{aligned} dn(t) &= [\theta_n(t) - a_n n(t)]dt + \sigma_n dW_n(t) \\ dr(t) &= [\theta_r(t) - \rho_{r,I}\sigma_I\sigma_r - a_r r(t)]dt + \sigma_r dW_r(t) \\ dI(t) &= I(t)[n(t) - r(t)]dt + \sigma_I I(t) dW_I(t) \end{aligned}$$

- Calibration on either YoY Cap / Floor or ZC Cap / Floor.
- Monte Carlo large steps model implementation.

3.3 Heston local volatility on Inflation Index

The dynamic of the inflation index is:

$$\begin{aligned} \frac{dI(t)}{I(t)} &= (n(t) - r(t))dt + \sqrt{V(t)} L(t, S(t)) dW_S(t) \\ dV(t) &= \kappa[\theta - V(t)]dt + \eta(t)\sqrt{V(t)} dW_V(t), V(0) = 1 \\ dW_S(t) dW_V(t) &= \rho dt \end{aligned}$$

- Calibration on YoY Cap / Floor and ZC Cap / Floor.
- Monte Carlo small steps model implementation
- See Stochastic Local Volatility models for Inflation

4 Foreign exchange

4.1 Static replication

The replication price is:

$$Price(t,T,S) = \gamma ZC(t,T) + \sum_{i=1}^{n} \alpha_i Call(t,T,S,F(t,T)+i\delta K) + \sum_{j=1}^{m} \beta_j Put(t,T,S,F(t,T)-j\delta K)$$

- Only available for vanilla contracts.
- Automatic decomposition.
- Quasi-Closed form solution.

4.2 FX Option Closed Form

4.3 Garman-Kohlhagen

The FX rate dynamic is:

$$\frac{dX(t)}{X(t)} = [r^d(t) - r^f(t)]dt + \sigma(t)dW(t)$$

Handle all Black Scholes features, including smart adjusters

4.4 Hull-White 1 factor + Garman-Kohlhagen

The dynamic under the domestic (cash) risk neutral probability of the FX rate is:

$$\begin{aligned} \frac{dX(t)}{X(t)} &= [r^d(t) - r^f(t)]dt + \sigma_X dW_X(t) \\ dr^d(t) &= [\theta_d(t) - a_d r^d(t)]dt + \sigma_d dW_d(t) \\ dr^f(t) &= [\theta_f(t) - \rho_{f,X}\sigma_X\sigma_f - a_f r^f(t)]dt + \sigma_f dW_f(t) \end{aligned}$$

- Interest rates are modeled with a Hull-White 1-factor model.
- Monte Carlo large steps or PDE model implementation.

4.5 Heston - Local Stochastic Volatility

The FX rate dynamic is:

$$\begin{aligned} \frac{dX(t)}{X(t)} &= (r^d(t) - r^f(t))dt + \sqrt{V(t)}L(t,X(t))dW_X(t) \\ dV(t) &= \kappa[\theta - V(t)]dt + \eta\sqrt{V(t)}dW_V(t), V(0) = 1 \\ dW_X(t)dW_V(t) &= \rho dt \end{aligned}$$

- Calibration to ATM volatility, Risk Reversal and Butterfly quotes using Heston calibration, Local Volatility calibration followed by either Particle Method or Fokker-Planck equation to compute probability density.

- Possibility to give more importance to Local or Stochastic volatility part using mixing weight factor.
- Advanced time discretization scheme for the variance (quadratic exponential).
- Monte Carlo multiple-asset or PDE model implementation.

5 Commodities

5.1 Static replication

The replication price is:

$$Price(t,T,S) = \gamma ZC(t,T) + \sum_{i=1}^{n} \alpha_i Call(t,T,S,F(t,T)+i\delta K) + \sum_{j=1}^{m} \beta_j Put(t,T,S,F(t,T)-j\delta K)$$

- Only available for vanilla contracts.
- Automatic decomposition.
- Quasi-Closed form solution.

5.2 Schwartz 1 factor

The dynamic of the commodity is:

$$\frac{dS(t)}{S(t)} = \kappa[\mu + g(t) - \log S(t)]dt + \sigma dW(t)$$

- Calibration on Futures, Calls on Spot and Calls on Future.
- Handle seasonality.
- Monte Carlo large step or PDE model implementation.

5.3 Schwartz 2 factors

The dynamic of the commodity and its convenience yield are:

$$\begin{aligned} \frac{dS(t)}{S(t)} &= (r(t) - c(t))dt + \sigma_S dW_S(t) \\ dc(t) &= \kappa(\alpha - c(t))dt + \sigma_c dW_c(t) \\ dW_S(t)dW_c(t) &= \rho dt \end{aligned}$$

- Calibration on Futures, Calls on Spot and Calls on Future.
- Monte Carlo large step or PDE model implementation.

5.4 Gabillon

The dynamic of the commodity is:

$$\frac{dS(t)}{S(t)} = \kappa \log\left(\frac{L(t)}{S(t)}\right) dt + \sigma_S dW_S(t)$$
$$\frac{dL(t)}{L(t)} = \sigma_L dW_L(t)$$

- Calibration on Futures, Calls on Spot and Calls on Future.
- Monte Carlo large step or PDE model implementation.

5.5 Clewlow-Strickland 1 factor

The dynamic of the commodity forward is:

$$\frac{dF(t,T)}{F(t,T)} = \sigma_t e^{-\alpha(T-t)} dW(t)$$

- HJM-like extension of Schwartz 1 factor.
- Take Future curve as input (and fit it exactly).
- Calibration on Calls on Spot and Calls on Future.
- Monte Carlo large step or PDE model implementation.

5.6 Clewlow-Strickland 1 factor Linear Stochastic volatility

The dynamic of the commodity forward is:

$$\begin{aligned} dF(t,T) &= \sqrt{V(t)}\sigma_r(t)e^{-\alpha(T-t)}dW_F(t) \\ \sigma_r(t) &= \sigma(t)\big[b_t F(t,T) + (1-b_t)F(0,T)\big] \\ dV(t) &= \kappa[\theta - V(t)]dt + \eta\sqrt{V(t)}dW_V(t), V(0) = v_0 \\ dW_F(t)dW_V(t) &= \rho dt1 \end{aligned}$$

- HJM-like extension of Schwartz 1 factor and the Stochastic volatility model.
- Take Future curve as input (and fit it exactly).
- Calibration on Spot and Future smile.
- Monte Carlo small step.

5.7 Forward curve building

The parametric yield is:

$$\begin{aligned} y_{a,b_1,b_2,c,\kappa,t_0}(t) = c + (1 - e^{-\kappa t}) &+ \frac{b_1}{2\pi}\big[\cos(2\pi(t-t_0)) - \cos(2\pi(t-t_0))\big] \\ &+ \frac{b_2}{2\pi}\big[\cos(4\pi(t-t_0)) - \cos(4\pi(t-t_0))\big] \end{aligned}$$

The parametric forward curve is:

$$F_{a,b_1,b_2,c,\kappa,t_0}(0,t) = S(0)e^{\int_0^t r(s)ds - y_{a,b_1,b_2,c,\kappa,t_0}(t)}$$

- Smooth forward curve building.
- Two seasonality effect (ie two first harmonics of seasonality effect).

6 Credit

6.1 Deterministic intensity

- Deterministic.
- Calibration on CDS spreads.
- Single risk.

6.2 Intensity with copula

- Multiple correlated risk factors.
- Calibration on CDS spreads and CDO tranches.
- Monte Carlo model implementation.

6.3 CDS , CDS Tranches pricer and CDS Swaption pricer

7 Hybrids

7.1 Generic hybrid: equity / interest rate / exchange rate / inflation / commodity

- Equities are modeled with a Black-Scholes model (with a term structure of volatility).
- Interest rates are modeled with a Hull-White 1 factor model.
- Exchange rates are modeled with a Garman-Kohlhagen model.
- Inflation indices are modeled with a Jarrow-Yildirim model.
- Commodities are modeled with a Clewlow-Strickland model.
- Credit are modeled with a intensity with copula model.
- Monte Carlo large steps model implementation.

7.2 Heston - Local Stochastic Volatility

- Handle hybrid FX / Equities / Inflation.
- Each underlying can have its own mixing weight factor (handle pure LV for asset and pure Heston for FX for instance).

7.3 Equity Local Volatility / Garman-Kohlhagen

- Hybrid equity / FX.
- Monte Carlo or PDE implementation.

7.4 Heston - Local Stochastic Volatility / Hull-White

- Handle hybrid FX / Equities / Interest Rate.
- Equities follow a Heston - Local Stochastic Volatility model.
- FX follow a Heston - Local Stochastic Volatility model.
- The short rate follows a Hull-White model with a volatility term-structure.
- Calibration with a particle method (exact calibration).
- Particular Heston / Hull-White and Local Volatility / Hull-White cases are supported

7.5 Hull-White 2 factors / Garman-Kohlhagen

- Handle hybrid FX / Interest Rate.
- Interest rates are modeled with a Hull-White 2 factor model.
- Exchange rates are modeled with a Garman-Kohlhagen model.
- Adapted to interest rate slope products (CMS spread), either hybrid, or quanto.
- Monte Carlo large steps model implementation.

7.6 Deterministic

- Hybrid all asset classes, using forwards for all underlying.

7.7 Priips

- LexiFi's Priips model implements most Priips approaches, as documented by regulation or encountered in the industry.
- Access to all intermediary results.
- Feeds LexiFi's customizable Priips document generation.

7.8 Time series

- Available for all asset classes
- Available in a uni-dimensional framework
  - AR(p)
  - ARMA(p,q)
  - GARCH(p,q)
  - MGARCH(p,q)
  - EGARCH(p,q)
  - AR(p)-GARCH(P,Q)
  - AR(p)-EGARCH(P,Q)
- Available in a multidimensional framework (Constant Conditional Correlation)
- Calibration using BFGS, Differential Evolution, Automatic Differentiation (AD)
- Monte Carlo implementation
- Historical probability simulations, with physical probabilities instead of risk-neutral ones

7.9 Multi dimensional static replication

$$
\begin{aligned}
Price_t &= \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_t^T r_u du}\text{Payoff}(S^D_{T_0},\dots,S^D_{T_N})\right] \\
&= \int_{\Omega^D_{T_N}} e^{-\int_t^T r_u du}\text{Payoff}(s^D_{T_0},\dots,s^D_{T_N})p^{\mathbb{Q}}_{S^D_{T_0},\dots,S^D_{T_N}}(s^D_{T_0},\dots,s^D_{T_N})ds^D_{T_0}\dots ds^D_{T_N}
\end{aligned}
$$

- Available for all asset classes
- Available for multiple fixing dates and underlyings
- A Monte Carlo is used to compute the expectation
- Use the algebraic representation of contracts to decompose the payoff in cashflows, options and conditions

The joint density is approximated using

- The marginals quoted on the market (european option)
- An hypothesis on the correlation matrix
- An hypothesis on the forward smile